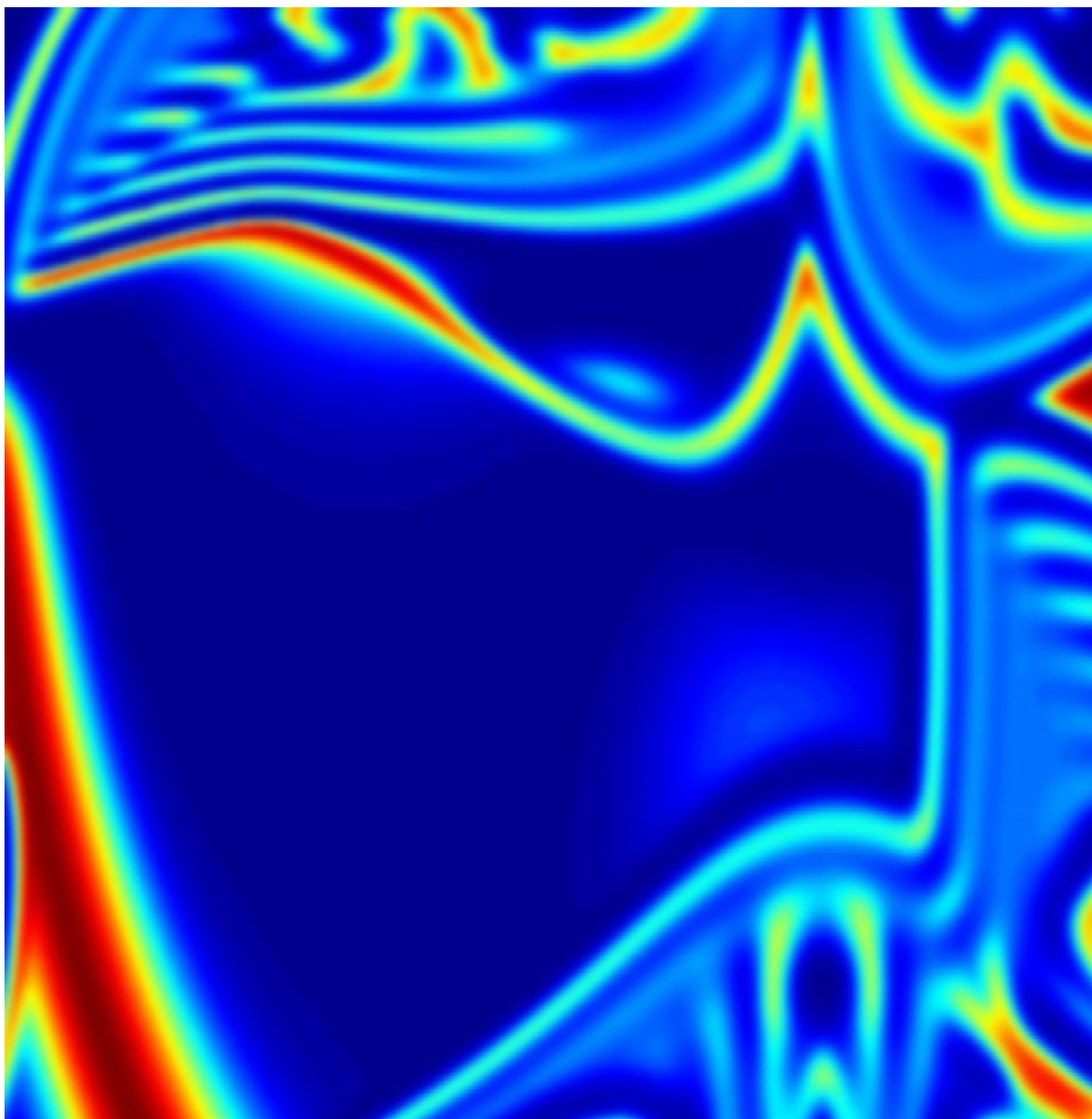


# MUDDmath

Harvey Mudd College

Volume 4, Issue 1 ■ Spring 2005



## Greetings and Salutations



Dear HMC Math Alums:

So much has changed since I joined the HMC faculty in 1989. When I arrived, HMC had about 560 students. Now it has nearly 700. Back then, the mathematics faculty consisted of myself, Bob Borrelli, Stavros Busenberg, Courtney Coleman, John Greever, Mel Henriksen, Hank Krieger, Alden Pixley, Mike Townsend and Al White.

Today, my colleagues are almost 90 percent different (listed by date of hire): Hank Krieger, myself, Lisette de Pillis, Weiqing Gu, Francis Su, Lesley Ward, Andrew Bernoff, Mike Raugh, Michael Orrison, Jon Jacobsen, Alfonso Castro (current chair of the department) and Darryl Yong '96.

But one thing has not changed during this time (aside from having great students, of course). We have not sent out a single newsletter. Of course, with all of the retirements and hiring, we have been very busy. But the time has come to change that.

So what have been the major changes during this time? Well, for one thing, the major has changed. Beginning in 1998, we replaced the various math tracks (General, Applied, Prob/Stat/OR, Computer Science) with a single unified major. Now all math majors take Discrete Math, Algebra, Analysis, Probability, Applied Analysis, a computational course, at least three math electives, and a year of Clinic or Thesis. Working with other departments, we created in 1998, a joint major in Computer Science and Mathematics and, in 2001, a major in Mathematical Biology.

The core mathematics sequence, required of all HMC students, has changed as well. Instead of four semester-long courses, we now offer eight half-semester-long courses, typically taken in the following order: Calculus of a Real and Complex Variable, Linear Algebra & Discrete Dynamical Systems, Differential Equations I, Multivariable Calculus I, Multivariable Calculus II, Probability and Statistics, Linear Algebra II and Differential Equations II. The last four courses are normally taken during the sophomore year, but we also offer them in an intensive six-week Summer Math program, which has been very popular with the students. This summer, over 90 students (half the freshman class) took summer math, taught by Professors Gu, Orrison, Jacobsen and Greg Levin '92.

During this time, we have also seen impressive growth of our major from a low of 10 majors in 1993 to a high of 42 majors (including joint majors) in 2002. Nowadays, we are seeing about 25 to 30 majors per graduating class, about half of whom are pursuing a double major or joint major. Nearly 50 percent of our majors have entered Ph.D. programs, many with prestigious fellowships, including one Rhodes Scholar (Elisha Peterson '99). In 2002, Josh Greene won the Morgan Prize for outstanding undergraduate research. Aaron Archer was runner-up for this prize in 1998.

From 1991 to 1994, we graduated only three women math majors. Since hiring Lisette de Pillis (1993), Weiqing Gu (1995) and Lesley Ward (1997) that number has grown dramatically. Nowadays, about one-third of our majors are women, and we hope that the trend continues. We have had three students receive runner-up or honorable mention for the Alice T. Schafer prize for excellence in mathematics by an undergraduate woman, awarded by the Association for Women in Mathematics.

All of these changes have occurred under the chairmanship of Bob Borrelli (1979–1990), Hank Krieger (1990–1995), Michael Moody (1996–2002), Art Benjamin (2002–2004) and Alfonso Castro (2004–present). Of course, the real power behind the thrones has been our administrative aides Sue Cook, Selina Zerbel, and (currently) Suzanne Frantz. And we would not be able to work nearly as productively without our Systems Administrator, Claire Connelly, hired in 2002.

The Mathematics Clinic Program has been successfully run by Bob Borrelli (1993–1999) and Michael Raugh (1999 to present), assisted by Barbara Schade since 1990. Michael Raugh also serves as director of the Research in Industrial Projects for Students (RIPS) program at the Institute for Pure and Applied Mathematics (IPAM). The RIPS program is based on the HMC Mathematics Clinic Program, and Bob Borrelli serves on the board of governors at IPAM.

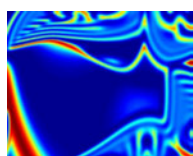
Since 1993, the department has resided in the Olin Science Center. Our departmental mathematics library boasts more than 2,000 books (thanks largely to donations from Professors Ives and White). Last year, the entire Claremont Mathematics Library collection was moved to HMC's Sprague library (which also has all of the Engineering and CS books). In exchange, the entire chemistry, physics and biology collections are now at Pomona's Science library. The department continues to cooperate with the other Claremont mathematics departments, sharing upper-division courses, colloquia and research seminars.

HMC students are familiar with the high quality of our teaching. Now the rest of the country is starting to see it as well. Francis Su was the first recipient of the Mathematical Association of America's Alder Prize for Distinguished

*(continued on page 13)*

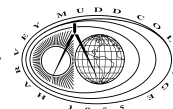
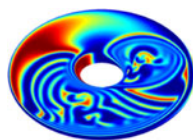
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## About the Cover

The cover image represents the spatial distribution of prey (phytoplankton) in a spatiotemporal model for plankton dynamics. The model consists of a system of reaction-diffusion equations whose reaction terms model classical predator-prey dynamics between the zoo- and phytoplankton. The diffusion term models mixing from turbulent ocean currents. Of particular interest are self-assembling patterns and patchy regions of species concentration. In joint work with undergraduate Jeff Hellrung '05, and Larry Li (UC Riverside, Dept. of Botany and Plant Sciences), Professor Jon Jacobsen studies pattern dynamics for such models on domains that are curved and/or dynamically growing. The goal of the study is to understand the effects that growth and curvature have on pattern formation and selection. The image shown represents a simulation on the surface of a torus. This work is supported in part by HMC's Center for Quantitative Life Sciences and the Beckman Faculty Research Program.



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Harvey Mudd College is a coeducational liberal arts college that seeks to educate scientists, engineers, and mathematicians, well versed in all of these areas and in the humanities and the social sciences, so that they may assume leadership in their fields with a clear understanding of the impact of their work on society.

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## HMC Places Fifth on Putnam Competition

The results of the nationwide 2003 William Lowell Putnam Mathematical Competition (held on Dec. 6, 2003) were announced in early 2004. Fifty-six HMC students chose to take this very hard six hour exam, which requires a unique blend of cleverness and problem-solving skills. Nationwide 3,615 students competed, and the median score this year was one out of a total of 120 points.

The HMC team of Andrew Niedermaier '04, David Gaebler '04, and Jason Murcko '04 won *fifth place* out of 479 schools! Each team member received a medal and a cash prize of \$200. Additionally the college received a \$5,000 prize. The top teams in the country this year were:

1. MIT
2. Harvard
3. Duke
4. Caltech
5. Harvey Mudd College.

Following us, in sixth through 10th place (in unannounced order): Princeton, University of British Columbia, UC Berkeley, University of Toronto, University of Waterloo. The HMC team has landed in the Top 10 in each of the past four years, but only once before has a Mudd team landed in the Top Five (third place, 1991). Actually, HMC is the only undergraduate college to have landed in the Top Five in the past 30 years, and we've now done it twice.

In the individual category, a total of seven students made the Top 200 List. Andrew Niedermaier '04 and David Gaebler '04 earned Honorable Mentions by placing in the top 50, and Jason Murcko '05, Jeffrey Hellrung '05, Eric Malm '05, Michael Vrable '04 and Robert Gaebler '04 all landed in the Top 200. A total of 20 students made the Top 500 List, including Melissa Banister '04, Ariel Barton '04, Jeff Brenion '05, Alan Davidson '06, Chris Erickson '06, Alex Eustis '06, Rachel Gabor '04, Duane Loh '04, Tyrel McQueen '04, Alex Popkin '04, Tyler Seacrest '06, Elan Segarra '06 and Lori Thomas '05. Only three other (much larger) schools could claim more students in the Top 500.



Putnam Before

## Three HMC Teams Win Top MCM Honors

HMC had another stellar performance in the 2004 International Mathematical Modeling Contest. Three HMC teams earned top honors of *Outstanding* and one team earned *Meritorious*.

A total of 21 HMC students participated in the 2004 competition. The contest gives each team of three 96 consecutive hours to develop a mathematical model and write a formal paper describing their work. The problems concerned: (A) the question of uniqueness of human fingerprints; (B) schemes for fast-pass systems at amusement parks; (C) optimizing security measures for academic information systems. There were 599 entries from around the world, of which only seven were deemed *Outstanding*, including our HMC team of Steven Avery '05, Eric Harley '04 and Eric Malm '05. This team, which chose Problem (A), also earned the SIAM Prize (Society for Industrial and Applied Mathematics) and the INFORMS Prize (Institute for Operations Research and the Management Science), both of which had an accompanying \$300 award per team member. The SIAM prize also supported travel for the team to present their work at the annual SIAM meeting.

For the Interdisciplinary Contest in Modeling (ICM), teams must choose Problem (C). From 143 entries worldwide, only four were selected as *Outstanding*, two of which were HMC teams: the team of Warren Katzenstein '04, Tara Martin '04 and Michael Vrable '04, and the team of Eli Bogart '05, Cal Pierog '05 and Lori Thomas '05.

All three outstanding teams will have their papers published in the *Journal of Undergraduate Mathematics and its Applications*. Other schools with outstanding papers include Harvard, University of Colorado at Boulder, University of Washington, Merton College at Oxford University (UK), and University College Cork (Ireland). Harvey Mudd College also had one team earning *Meritorious* (top 10 percent), composed of Les Fletcher '04, David Gleich '04 and Jeff Hellrung '05, and three teams earning *Honorable Mention* (top 25 percent).



Putnam After

### NSF Fellowships and NSF Honorable Mentions in 2003, 2004

Three mathematics majors from HMC have won National Science Foundation Graduate Research Fellowships this year, and seven more won Honorable Mentions. This represents more than a fourth of the senior math majors that are graduating this year.

Math majors David Gaebler '04, Andrew Niedermaier '04 and Anne Short '02 were awarded these prestigious fellowships, which provide \$30,000/year plus a \$10,500 cost-of-education allowance for three years of graduate study. Gaebler and Niedermaier's awards were in mathematics; and Anne Short's award was in environmental sciences. In addition seven HMC math majors won Honorable Mentions, including seniors Melissa Banister, Ariel Barton, Adam Bliss, David Gleich (CS/Math), Michael Vrable (CS/Math), and Chris Pries '02 and Ross Richardson '02.

In 2003, Melissa Chase (double major math/CS), Nate Eldredge (math), Andrew Iannaccone (math), and Daniel Lowd (joint CS-math) were awarded NSF Fellowships, and math majors Adam Bliss, Christopher Pries, Ross Richardson, Jeremy Rouse, Micah Smukler and David Uminsky received honorable mentions. Jeremy Rouse '03 also won an National Defense Science and Engineering Fellowship, which awards \$23,500/year for three years of graduate study.

### Ariel Barton Wins Honorable Mention for Schafer Prize

Ariel Barton '04 was awarded Honorable Mention for the Alice T. Schafer Prize for excellence in mathematics by an undergraduate woman, awarded by the Association for Women in Mathematics (AWM) at the AMS-MAA Winter Meetings in Phoenix in January 2004. The criteria for selection include, but are not limited to, the quality of the nominees' performance in mathematics courses and special programs, an exhibition of real interest in mathematics, the ability to do independent work, and if applicable, performance in mathematical competitions.

### Eric Malm Wins National Problem Solving Competition

Senior mathematics major Eric Malm took first prize at the National Problem Solving Competition, sponsored by the Mathematical Association of America, held at MathFest in Providence, RI, on Aug. 14, 2004. Eric was the first student (among 25 participants, representing 20 institutions) to correctly solve seven challenging math problems on topics including geometry, number theory, combinatorics, probability and differential equations. Richard Neal, director of the competition, said that Eric "finished the exam in record time." Eric needed less than 30 minutes to answer all seven questions. Here is one of the questions:

Without using a calculator, find the prime factorization of  $4^9 + 3^9$ .

For his first place finish, Eric received a paid summer internship at Lawrence Livermore Labs. Also participating in the competition was math major Eric Harley '04. Since the Problem Solving Competition's inception in 1998, several HMC students have finished in second place: Andrew "Rif" Hutchings '98, Paul SanGiorgio '01, Daniel Boylan '02 and Robin Baur '06.

### Tara Martin Wins Watson Fellowship

Tara Martin '04, a mathematical biology major, won a Thomas J. Watson Fellowship for the 2004–2005 academic year. Tara's project, "Finding the Inner Beat: Cultural Expression Through Movement," will take her to Argentina, Brazil and Cape Verde as she studies dance forms including tango, samba, capoeira and their African precursors.

### Ben Nahir Wins Award at Sigma Xi Student Research Conference

Ben Nahir '04, a mathematical biology major, won the Superior award for best poster in Immunology-Neuroscience at the November 2003 National Sigma Xi Student Research Conference in Los Angeles.

Ben's project, "Cloning, Localization, and Characterization of Tolloid and BMPR-II in *Aplysia californica*," was co-authored by Sean Reagin (University of Georgia), and Andrea Kohn, Thomas Ha and Leonid Moroz (all from the University of Florida).

### ClinicProjects 2003–04

#### Solitons in Shallow Water Waves

**Sponsor:** Los Alamos National Laboratory

**Team Members:** Lindsay Crowl  
Kevin Andrew  
Christian Bruun  
Jon Goldis

**Advisor:** Alfonso Castro

**Abstract:** Los Alamos National Laboratory is currently researching various properties of nonlinear shallow water wave equations. With the help of the Clinic's liaison, Dr. Darryl Holm, the team analyzed the behavior of certain solutions to the Camassa-Holm equation. Their research included both a theoretical and a numerical analysis of soliton-like shallow water waves called peakons. For the numerical investigation, the team created a numerical integrator for the third order, nonlinear Camassa-Holm equation. The theoretical results were compared to numerical simulations that visualize various aspects of wave behavior in both the one- and two-dimensional cases.

#### Study of a Geo-Location System

**Sponsor:** Sandia National Laboratories

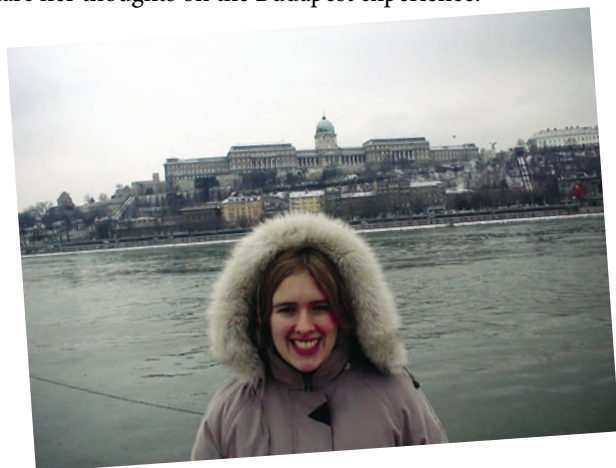
**Team Members:** Andrew Niedermaier  
Todd Cadwallader-Olsker (CGU)  
Luke Finlay (Fall, visiting from UniSA)  
Elizabeth Millan  
Josh Padgett

**Advisor:** Weiqing Gu

**Abstract:** In updating the current search-and-rescue satellite system to a geosynchronous, multi-satellite system, we can now detect a distress beacon instantaneously anywhere on the earth. The team's task was to implement an algorithm that takes the data received by multiple satellites and quickly determines the location from which the original message was sent.



Last Spring, Melissa Banister participated in the Budapest Semesters in Mathematics Program. MUDDmath asked Melissa to share her thoughts on the Budapest experience.



“Going to Budapest was one of the best decisions I have ever made, short of my decision to enroll at HMC, of course! The program provides the perfect balance between freedom and guidance, between mathematics and adventure. My arrival in Budapest found me exhausted, uncertain, and more than a little anxious. I had arrived a few days before the start of the program and so had several days to become comfortable in the city before class began. Even though I had a little culture shock at first, I soon found that Budapest is a friendly, safe city. Hungarian is a challenging and unique language, but it is perhaps this uniqueness that makes vocabulary words stick so clearly in one’s head. And most of the younger people speak English to some degree, so I was rarely unable to communicate on a basic level with those around me.

“The mathematics courses in the program were challenging, rich and exciting. When your professor has an Erdos number of 1, you definitely want to make sure you follow his combinatorics lecture carefully! My favorite course was Number Theory, taught by the energetic Csaba Szabo, who somehow found time to be quite the comedian during our fast-paced lectures. A typical Friday afternoon might consist of our Number Theory class going to Margit Island to work on a problem set, followed by a game of Ultimate Frisbee with the whole gang. Then, off to a delicious, rich, and lard-based Hungarian dinner in Moscow Square, followed, perhaps, by a \$2.50 movie!

“Somehow, amidst all this crazy mathematics, we found time to travel. Many famous cities are just a relatively short train ride away, and train rides are a great time to work on conjecture and proof problem sets. I personally went to Poland, Romania, the Czech Republic, Slovakia, Slovenia, Croatia, Italy and Austria. It was easily the richest and most amazing semester of my life, and any budding mathematician with a taste for graph theory and goulash should definitely take advantage of this amazing opportunity.”

Recent Mudders to spend a semester in Budapest included Sarah Mann ’06, Nick Rauh ’06, Ariel Barton ’04, Dave Gaebler ’04, Andy Niedermaier ’04, Matt Macauley ’03 and Jeremy Rouse ’03.

## Senior Theses 2004

### Melissa Banister

Separating Sets for the Alternating and Dihedral Groups

Advisor: Michael Orrison

### Ariel Barton

Conditions on Harmonic Measure Distribution Functions of Planar Domains

Advisor: Lesley Ward

### William Chang

Image Processing with Wreath Products

Advisor: Michael Orrison

### Grant Clifford

A Decomposition of Voting Profile Spaces

Advisor: Jon Jacobsen

### Robert Gaebler

Alexander Polynomials of Two-Bridge Knots and Links

Advisor: Jim Hoste (Pitzer)

### David Gaebler

Toeplitz Operators on Locally Compact Abelian Groups

Advisor: Hank Krieger

### Eric Harley

Mathematically Modeling Spatial Tumor Growth in Parallel

Advisor: Lisette de Pillis

### Matthew Holden (Pomona)

Volume Minimizing Cycles in  $G_2$ -manifolds

Advisor: Weiqing Gu

### Jessica Nelson

On the Erdős Problem of Empty Convex Hexagons

Advisor: Lisette de Pillis



Senior Thesis Students (l to r) with Thesis Coordinator Lisette de Pillis (second from right): Melissa Banister, Rob Gaebler, Will Chang, Ariel Barton, Jessica Nelson, Grant Clifford, Eric Harley, Dave Gaebler and Matt Holden.

## Park City Mathematics Institute

July 2003 took Professors Lesley Ward and Andrew Bernoff to the Park City Mathematics Institute (PCMI), sponsored by the Institute for Advanced Study. Park City is located in the mountains just outside Salt Lake City, Utah, and is best known to most people for hosting the 2002 Winter Olympics. But your typical mathematician on the street knows it is home to one of the premiere mathematics summer schools. The PCMI program brings together undergraduate and graduate students, high school educators, college faculty, and researchers to exchange information and insight about mathematics.

In 2003, the program theme was Harmonic Analysis. Lesley taught a course on wavelets, which was aimed at undergraduates but which also interested a number of graduate students. Andy taught the Undergraduate Faculty Program, designed to teach faculty how to assemble an introductory PDEs course using computational tools such as MAPLE. Other Claremont mathematics professors in attendance included Darryl Yong '96, Jon Jacobsen and Jorge Aarão (CMC). Jon gave his world-famous soap bubble demonstration, and Darryl showed that timpani drums can double as a demonstration of the wave equation. Andy's course also discussed tips and techniques for mentoring undergraduate research—Jeff Hellrung '05 presented the results of his research with Professor Jacobsen and expounded upon the joys and frustrations of undergraduate research.

A highlight of the program was a Putnam Problem Solving-style seminar attended by a broad range of PCMI participants, including high school teachers, undergraduates, graduates and the occasional research mathematician. Pizza and desserts, as always, were a hit among the participants.

In 2004, PCMI turned to Geometric Combinatorics, (Q: How many hyperplanes does it take to screw in a light bulb? A: Three, but only if they are in general position.), where Professor Francis Su continued HMC's ongoing participation in the program. More details can be found at

<http://www.admin.ias.edu/ma/>.

## Inaugural Teaching Award Goes to Professor Su

Francis Su received the 2004 Henry L. Alder Award for Distinguished Teaching by a Beginning College or University Mathematics Faculty Member. Awarded for the first time in 2004 by the Mathematical Association of America (MAA), the honor goes to college or university faculty who have taught full time for between two and seven years, whose teaching has been extraordinarily successful and whose effectiveness in teaching undergraduate mathematics has had influence beyond their own classroom. Francis and the other awardee, Mills College professor Zvezdelina Stankova, received \$1,000 and a certificate of recognition from the MAA at MathFest 2004 in Providence, Rhode Island.

Francis, an HMC faculty member since 1996, is known for his creative teaching strategies and engaging teaching style in the

wide variety of courses that he teaches. He is one of the coaches of HMC's William Lowell Putnam Mathematical Competition team, a program he helped grow from 20 to about 70 students. The HMC team has earned a top 10 spot every year for the last three years, and this year's fifth place finish is the best by the college since 1991. HMC is the only undergraduate college to have placed in the top five in the past 30 years of the Putnam Competition (see page 2).

Each year, Francis co-authors papers with several undergraduate students, most of whom go on to graduate study at top institutions. Faculty across the United States have used his Web site "Mudd Math Fun Facts" ([www.math.hmc.edu/funfacts](http://www.math.hmc.edu/funfacts)) to show students that mathematics is full of interesting ideas, patterns and new modes of thinking. He has designed conferences and seminars for Project NExT, a program for new or recent Ph.D.s in the mathematical sciences who are interested in improving the teaching and learning of undergraduate mathematics.

Francis spent the Fall 2003 semester as a member of the Mathematical Sciences Research Institute in Berkeley, California, participating in the fall program on "Discrete Geometry." The program focused on problems at the intersection of geometry and combinatorics, including problems on polytopes. His recent research with HMC students has focused on triangulations of polytopes and applications to mathematical economics. In addition to his research obligations, Francis also hosted an unusual seminar for Berkeley undergrads to introduce them to the research at MSRI. In May 2004, he returned to MSRI to run a workshop on geometric combinatorics.

## Model Professors

Robert Borrelli and Courtney Coleman, both professors of mathematics emeritus, have published the second edition of their textbook "Differential Equations: A Modeling Perspective" (John Wiley & Sons, Inc.). The book teaches differential equations by using them to create models to solve real-world problems. To aid computer-based learning, the book comes with a CD that includes ODE Toolkit 1.5, which was developed in part at HMC. Jennifer Switkes '94 wrote the three technology resource manuals that accompany the teacher's edition and that explain how to use the Maple, Matlab and Mathematica mathematics programs on the CD.



Courtney Coleman and Robert Borrelli with the latest edition of their book.

### A Note from Mel Henriksen

As some of you know, I taught and was active in research in mathematics here at HMC from 1969 to 1997. Actually, my love affair with learning mathematics and passing it on to others through teaching and research goes back to 1951. While I no longer do any classroom teaching, I come to the campus to pursue my scholarly activities almost every day except when I am traveling. So, if any of you wish to see me, I will probably be available as well as eager to see you and talk about the halcyon days when you were here. You can find me on the fourth floor of the Sprague Library. You may also send me e-mail at [henriksen@hmc.edu](mailto:henriksen@hmc.edu) to renew acquaintance or to discuss mathematical matters.

### Benjamin Update

The Mathematical Association of America recently published "Proofs that Really Count: The Art of Combinatorial Proof" by Arthur Benjamin and Jennifer Quinn. Art and Jennifer are also the co-editors of *Math Horizons* magazine, published by the MAA. Art was just elected to serve a three year term as governor of the Southern California-Nevada Section of MAA. He is spending the 2004–05 academic year on sabbatical at Brandeis University near Boston and the University of New South Wales in Sydney, Australia.

### Ward Update

Prof. Lesley Ward spent her Spring 2004 sabbatical at the Australian National University in Canberra, where she held the WEIS Visiting Fellowship for a Senior Academic Woman in Engineering and Information Sciences, and was able to pat some kangaroos. As well as completing two articles co-authored with HMC students in the areas of complex analysis and Web search, she began a new research project in the area of harmonic analysis with collaborator Jill Pipher of Brown University. Ward visited the University of South Australia in Adelaide to work with Jerzy Filar (who spent the spring of 2002 at HMC) and Vladimir Ejov on some applications of complex analysis to the stock market. She also visited the University of New England in Armidale. At the ANU, Ward taught the second-year honours level real analysis class. As the WEIS Visiting Fellow, she participated in a peer support group for women faculty, set up a group of women graduate students who took turns giving technical talks, and gave a demonstration workshop for students who will be visiting high schools to promote technical careers.

In May 2004, Ward taught an undergraduate course entitled "Harmonic Analysis: from Fourier to Haar" at the IAS/Princeton University Program for Women in Mathematics, with Cristina Pereyra of the University of New Mexico. HMC student Lindsay Crowl '04 was one of the participants.

### Bernoff Update

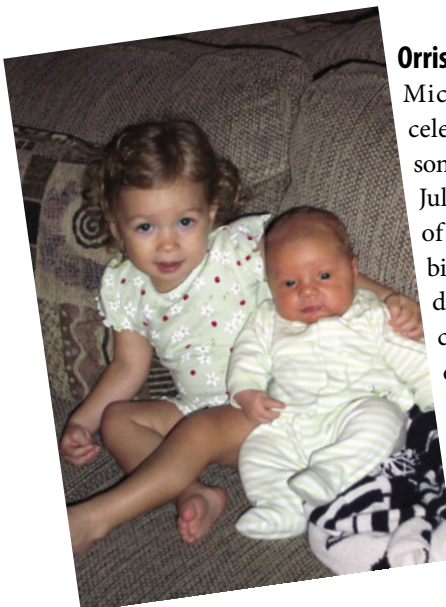
Prof. Andrew Bernoff spent part of Spring 2004 on sabbatical at Berkeley, Boston University and the University of Cambridge in England. However, his big news is that on February 18, 2004 he married his long-time partner, Thomas Trautmann, in San Francisco, becoming one of the four thousand gay and lesbian couples married by the city that spring. In anticipation of moving to San Francisco for most of the semester, they bought a bright yellow Mini Cooper. Prof. Bernoff's favorite man and machine can both be seen in the picture below.



On the mathematical front, his sabbatical was spent delving into the mysteries of self-similarity and scaling behavior in fluid dynamical systems, in particular micro-fluids. Part of his time was spent deriving models to describe the behavior of lipid domains on a fluid substrate (think of a little blob of fat on top of a bowl of chicken soup, only at the micron-scale) in collaboration with an experimental group at Case Western Reserve University. He also has been studying the thin film equation, which models the de-wetting of a substrate, and the production of micron-scaled aerosols (with Prof. Tom Donnelly, Physics).

### Orrison Update

Michael and Jody Orrison celebrated the birth of their son Ethan Douglas Orrison on July 9, 2004. Here is a picture of Ethan relaxing with his big sister Sydney. When not doing mathematics, both children enjoy singing, dancing, and laughing with mama and papa.







**Arthur Benjamin**, *Professor* (Ph.D., Johns Hopkins University): combinatorics, game theory, and graph theory; operations research.



**Andrew Bernoff**, *Professor* (Ph.D., Cambridge University): applied mathematics, fluid mechanics, free boundary problems, self-similarity.



**Alfonso Castro**, *Kenneth A. & Diana G. Jonsson Professor and Department Chair* (Ph.D., University of Cincinnati): partial differential equations; nonlinear functional analysis.



**Lisette de Pillis**, *Professor* (Ph.D., University of California, Los Angeles): numerical linear algebra; computational fluid dynamics; mathematical biology.



**Weiqing Gu**, *Associate Professor* (Ph.D., University of Pennsylvania): differential geometry and topology; optimal control; geometric modeling; computer-aided geometric design.



**Jon Jacobsen**, *Assistant Professor* (Ph.D., University of Utah): nonlinear analysis, partial differential equations, mathematical biology.



**Henry Krieger**, *Professor and R. Stanton Avery Fellow in Mathematics* (Ph.D., Brown University): probability theory and stochastic processes; Toeplitz operators.



**Michael Orrison**, *Assistant Professor* (Ph.D., Dartmouth College): algebra, combinatorics, computational non-commutative harmonic analysis.



**Michael Raugh**, *Professor and Director of the Mathematics Clinic* (Ph.D., Stanford University): scientific computing; mathematical modeling.



**Francis Su**, *Associate Professor* (Ph.D., Harvard University): topological and geometric combinatorics, fair division and mathematical economics.



**Lesley Ward**, *Associate Professor* (Ph.D., Yale University): complex analysis, Web search, harmonic analysis.



**Darryl Yong '96**, *Assistant Professor* (Ph.D., University of Washington): perturbation theory, numerical analysis, ordinary and partial differential equations.



**Suzanne Frantz**, *Department Administrative Aide*



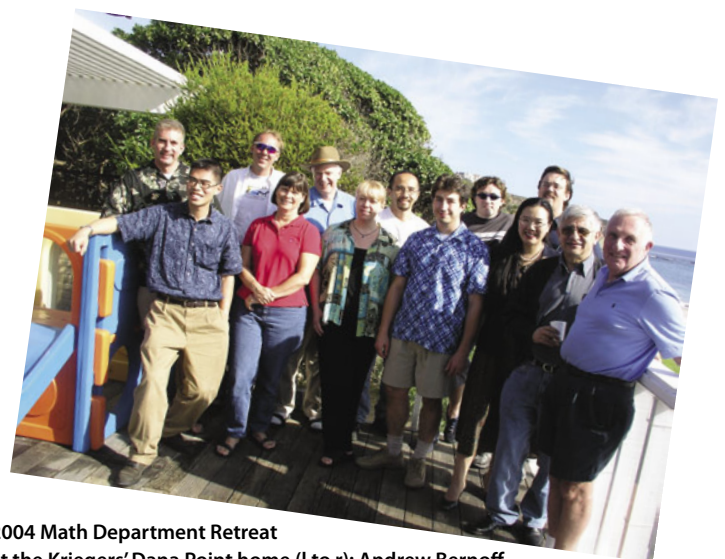
**Barbara Schade**, *Mathematics Clinic Administrative Aide*



**Claire Connelly**, *Systems Administrator*

## Visiting Faculty

Recent visitors to the department include John Milton (Chicago University, now at the Claremont Colleges) and Ed Packel (Lake Forest College) during the Fall 2003 semester, Claus-Jochen Haake (University of Bielefeld) during the Spring 2004 semester, Dann Mallet (now at Queensland University of Technology) in Spring and Fall 2004, and John Neuberger (Northern Arizona University) in the 2004–05 academic year.



**2004 Math Department Retreat**  
at the Kriegers' Dana Point home (l to r): Andrew Bernoff, Darryl Yong '96, John Neuberger, Suzanne Frantz, Michael Raugh, Lisette de Pillis, Francis Su, Michael Orrison, Dann Mallet, Weiqing Gu, Jon Jacobsen, Alfonso Castro, and Hank Krieger

## **Catching up with Michael Moody**

*Eric Harley '04 recently caught up with Michael Moody, former HMC Department of Mathematics chair, currently Dean of Faculty at Olin College in Needham, Massachusetts. Here's an excerpt of their conversation.*

### **How did you get involved with Olin?**

During Mudd's WASC accreditation review in 1999, the head of the team was the new president of Olin. He was on the look out for people with experience to bring there. Well, one thing led to another, and they asked me to help as a consultant.

### **How long have you been there?**

I started as Dean of Faculty in July of 2002. They had hired the first faculty members in 2000. As a consultant, I helped them find and hire some more faculty members. At the time, they didn't have any one to do it so they sought people who had experience to do it.

### **How are things at Olin these days?**

We're starting to feel the excitement of our first graduation. It'll happen in May 2006. There are 68 students graduating out of the original 75.

### **It must be rewarding to be involved with Olin at this stage.**

Yeah, to see my workload grow exponentially! Ah, the good old days when we only had freshman around here. It's become a very busy place from my personal point of view. We've been adding 5 or 6 new faculty members every year. We're going to 300 students total in the next year. I really miss the people at Mudd. It was really difficult to come to Olin. The opportunity presented itself. And you don't get many chances to create a school from the beginning. That doesn't mean the positive things I feel about Mudd have gone. It's still a hard place to have left.

### **Do you still have time for teaching?**

I teach more than I really should. I live on campus, so I manage to put in a lot of hours. I like teaching, that's how I meet the students.

### **Since there's only engineering, how does the quality and breadth of the students compare to Mudd?**

When the students come to Olin they know that the only degree is engineering. They don't necessarily know what that means, but the student body is really great. Their test scores are up there with Caltech, MIT and Harvey Mudd College. The average is 1500 on the SAT. Our selection process gets students in here that are pretty broad, multi-talented, and not single minded. Overall, most of the students are pretty well-rounded.

What we don't have here that one would find at Harvey Mudd College are certain kinds of prodigies, terrifyingly brilliant students, like freshman in abstract algebra. Olin doesn't have a curriculum for such a student. Occasionally, we'll get a student like that in the pool. We're not the right school for that student. They'll write how much they want to continue in, say math, and we're thinking that we don't have things like topology.

### **Are you still getting out there and partying it up amongst the student body?**

I haven't been to a single student party, although rumors have been spreading around Olin. I was asked the other day by some students, "Tell us about the Catholic school girls party." I didn't tell them about it. You know, I went out of my way not to go to that one, but I dropped by Linde one night and Colin Jemmot was tending bar.

### **When you left Claremont, your wife was in the middle of law school. How has that worked out for her?**

My wife finished law school and took the bar exam the end of last July. She's now working at a law firm in downtown Boston, doing a variety of things. It's a small, recently founded firm. She's been working on federal anti-trust cases, federal sentencing guidelines, and other lawsuits. I have to say that's one of the greatest things about our society. A 50-year-old woman can get a law degree and change her career, and everybody is supportive. Our educational system is filled with chances to do anything at any time. Most other countries, they'd say "You can't do that, that's not for you."

### **How's your hearty black lab, Maisy, adjusting to New England?**

Turns out she doesn't mind the snow. She enthusiastically gambols about when it snows. And we do live out in the western suburbs. There's a lot of wooded areas around us. She has managed to get into a skunk.

### **What have you been reading lately?**

*The Art of Eating*, by M.F.K. Fischer. It isn't a cook book. Occasionally there is a recipe, but it's really about the high level appreciation of the role of eating in a refined culture. Recently I finished a John Keegan book, *The Face of Battle*, but I haven't had a lot of time for other reading.

And I would just like to get it out there that anybody in the Boston area should stop by Olin and say hello.

## HMC Mathematics Conference Series

Since 1999, the Department of Mathematics has hosted an annual mathematics conference, with varying topics each year. These research-level conferences expose the faculty and students to cutting-edge mathematics and serve as a convenient gathering place for mathematicians and mathematics students from the Southern California area.

The conference topics to date, with local organizer(s), are:

1999	Analysis	(Prof. Ward)
2000	Applied Analysis	(Profs. Bernoff and Hosoi)
2001	Differential Geometry	(Prof. Gu)
2002	Applied Algebra & Combinatorics	(Prof. Orrison)
2003	Mathematical Biology	(Profs. de Pillis and Jacobsen)
2004	Geometry, Algebra and Phylogenetic Trees	(Prof. Su)

To help introduce a particular conference and its topic to the HMC community, conferences have featured bonus evening and/or class lectures from one of the invited speakers, just prior to the conference. For instance, Herman Gluck spoke to an advanced geometry class on "Vector calculus and the topology of domains in 3-space," Persi Diaconis presented "From shuffling cards to the roots of randomness" for a general audience of over 300, and Bard Ermentrout presented "Dynamics of sugar-water oscillators and drinking birds" to two freshman differential equation classes.

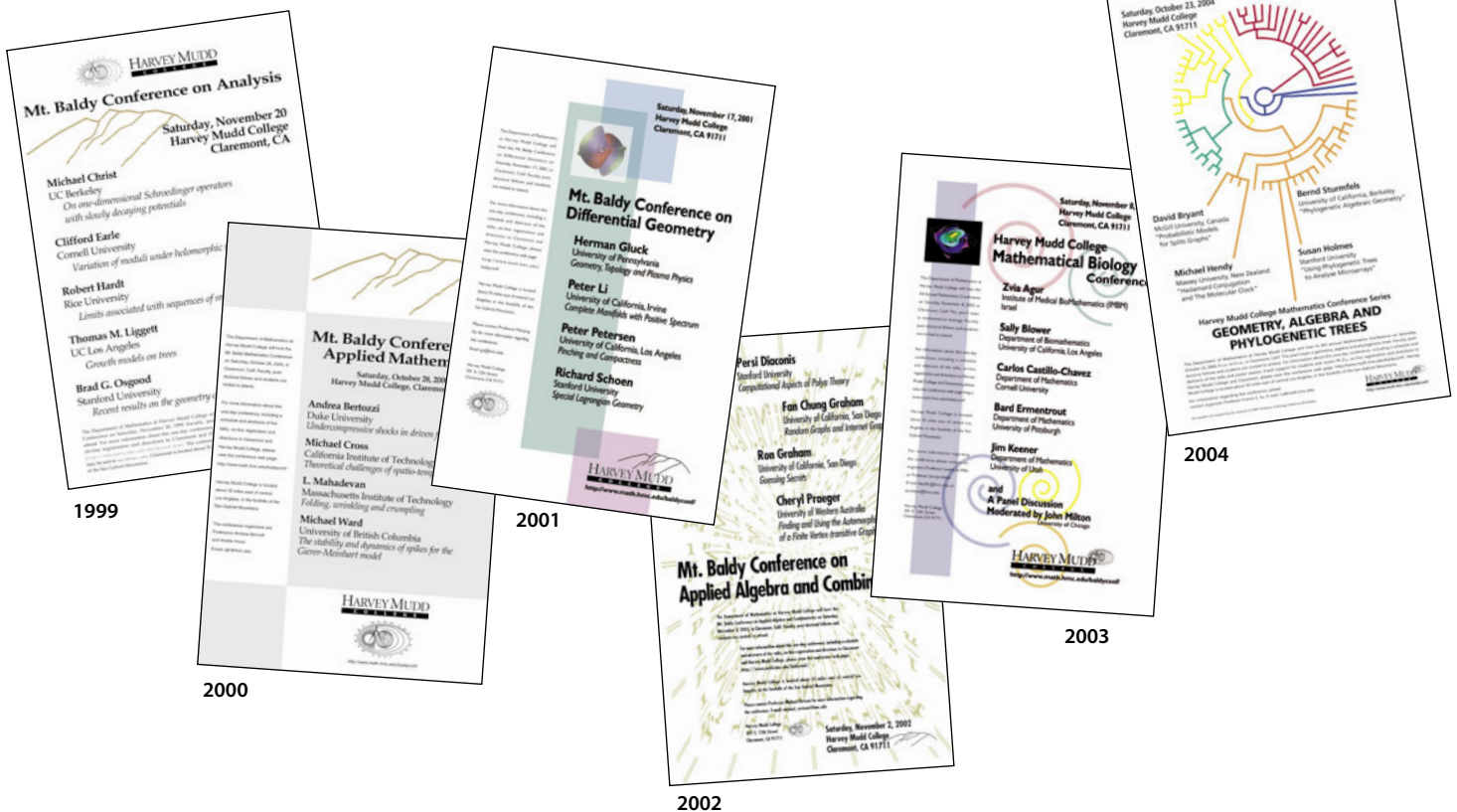
Bard Ermentrout was particularly thrilled when Bob Borrelli shared his 30-year-old "Happy Drinking Bird" (aka "Dippy Bird" or "Drinking Duck"), which worked remarkably well during his talk.



Left to Right: Jim Keener (University of Utah), John Milton (University of Chicago, now KGI & Joint Science), Sally Blower (UCLA), Zvia Agur (Institute of Medical BioMathematics, Israel), Bard Ermentrout (University of Pittsburg) and Carlos Castillo-Chavez (Cornell University).

Recently, we were pleased to learn that the NSF will support these conferences for 2004-06, allowing us to continue to host a high-level research conference on the HMC campus. The NSF support will also allow for a contributed poster session and provide travel support for five to ten graduate students and recent Ph.D.s to participate. Congratulations and thanks to Profs. Ward and Orrison for their hard work in securing the NSF funds.

More information on the HMC Mathematics Conference series can be found at <http://www.math.hmc.edu/baldyconf.html>.



**Neural Response at the Edge of Instability**

John Milton (University of Chicago)

**Aharonov and Bohm Meet Cauchy and Riemann in a Pigeonhole**

Michael Christ '77 (University of California, Berkeley)

**Projectile Motion with Resistance, Experimental Mathematics and the Lambert W Function**

Ed Packel (Lake Forest College)

**On the Metamorphosis of Vandermonde's Identity**

Don Rawlings (Cal Poly, San Luis Obispo)

**Erdős' Distinct Distances Problem**

Van Vu (University of California, San Diego)

**Identifying Sporadic Simple Groups from Odd Local Data**

Chris Parker (University of Birmingham)

**Invariants of Curves in the Projective Plane**

Abigail Thompson (University of California, Davis)

**Erdős-Szemerédi Sum Product Problem**

Mei-Chu Chang (University of California, Riverside)

**The A-Polynomial of a Knot**

Jim Hoste (Pitzer College)

**Recurrence and Topology in Dynamical Systems**

John Alongi (Cal Poly, San Luis Obispo, visiting POM)

**Finite Fields and Polynomials**

Greg Stein (City University of New York, visiting CMC)

**Has the Continuum Hypothesis Been Solved?**

Matt Foreman (University of California, Irvine)

**Beware: Counting Can Lead to Algebraic Geometry**

Aaron Bertram (University of Utah)

**The Number Theory of Partitions: The Legacy of Euler, Freeman Dyson and Ramanujan**

Ken Ono (University of Wisconsin, Madison)

**On Classifying Finite-Dimensional Hopf Algebras**

Susan Montgomery (University of Southern California)

**The Wonderful World of Composition**

Dylan Retsek (Cal Poly, San Luis Obispo)

**Digital Geometry Processing**

Peter Schröder (California Institute of Technology)

**Geometry, PDEs, CFD and Image Processing**

Tony Chan (University of California, Los Angeles)

**C-Symmetric Operators**

Stephan Garcia (University of California, Santa Barbara)

**Some Interesting Definite Integrals**

Victor Moll (Tulane University)

**The Classes of  $k$ -Hyponormal and Weakly  $k$ -Hyponormal Operators**

George Exner (Bucknell University)

**Superadditive Object Division or**

**"You Scratch My Back and I'll Scratch Yours"**

Claus-Jochen Haake (University of Bielefeld, visiting HMC)

**Using Computer Simulations to Reverse-Engineer Cilia**

Charles Brokaw (California Institute of Technology)

**Pieces of Ergodic Theory**

Florence Newberger (Cal State University, Long Beach)

**Counting on Determinants**

Art Benjamin (Harvey Mudd College)

**Orthogonal Polynomials and Sum Rules**

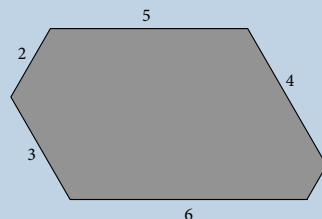
Barry Simon (California Institute of Technology)

**Partition Computations**

George Andrews (Pennsylvania State University)

**Bernoff's Puzzler**

Show that there are infinitely many positive integers  $N$  such that you can construct an  $N$ -sided equiangular polygon with sides of length  $1, 2, 3, \dots, N$ , not necessarily in that order. A polygon is equiangular if the angle between any pair of consecutive sides is the same. The first  $N$  for which this is possible is 6—the polygon could have sides  $1, 4, 5, 2, 3, 6$ .

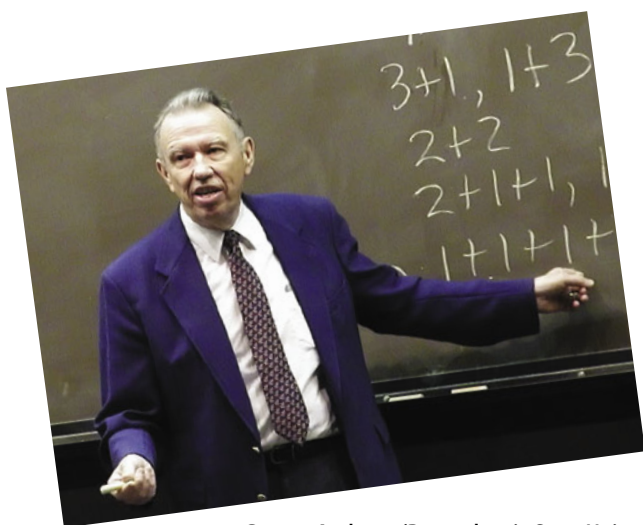


**Extra Credit:** Determine all values of  $N$  for which this is possible.

Send your answers to [bernoff@math.hmc.edu](mailto:bernoff@math.hmc.edu).



In addition to our weekly colloquia, the department hosts special evening mathematical talks by distinguished mathematicians, aimed at the entire HMC student body. Recent speakers have included George Andrews (Pennsylvania State University) and Helmer Aslaksen (National University of Singapore). Andrews, who was elected to the National Academy of Sciences in 2003, has long been interested in the work of Ramanujan, the Indian genius, whose last notebook he unearthed in the Trinity College Library at Cambridge in 1976. His fascinating talk explored the life of Ramanujan and his impact on mathematics. Aslaksen's lecture on the mathematics of calendars from different cultures was also delightful and curiously coincident with a lunar eclipse.



George Andrews (Pennsylvania State University) speaking on "The Lost Notebook of Ramanujan" on April 27, 2004. (Photo by Lee Jones)



Helmer Aslaksen (National University of Singapore) giving a lecture entitled "Heavenly Mathematics: The Mathematics of the Chinese, Indian, Islamic and Gregorian Calendarst" on October 28, 2004. (Photo by Lee Jones)

## Joint Major in Computer Science and Mathematics

The Joint Major in Computer Science and Mathematics is a program of study tailored to students who are interested in the interdisciplinary connections between computer science and mathematics. There are many areas of overlap between these two disciplines. For example, techniques from formal logic are used to prove rigorously that a computer program correctly performs a specified task. Complexity theory uses techniques from mathematics and computer science to determine precisely the difficulty of a computational problem. Numerical analysis examines methods for computing numerical solutions to a variety of mathematical problems which arise in areas ranging from medicine to aircraft design.

The joint major provides students with the foundations of mathematics and computer science, giving them a firm grounding in both the applied and theoretical aspects of these fields, and permitting them to explore those areas which they find most compelling. The required courses in this major are drawn from both the computer science and mathematics curricula, and there are many possible elective courses from which to choose.

Students who complete the Joint Major in Computer Science and Mathematics are well-prepared for immediate employment in a variety of areas including software, mathematical finance and consulting. The major also provides excellent preparation for graduate studies in mathematics, computer science, operations research and other related fields.

## Mathematical Biology Major

The current century has been predicted to be "the century of biology"; and as biology and biotechnology become more important in the coming decades, so will the application of quantitative methods to biological science. Mathematical and computational components are vital to many areas of contemporary biological research, such as genomics, molecular modeling, structural biology, ecology, evolutionary biology and systems analysis of neurobiology, physiology and metabolism. Students interested in the interface between biology and mathematics may pursue the Mathematical Biology major, which is jointly offered by the Departments of Biology and Mathematics.

The Mathematical Biology major prepares graduates for further study in either biology or applied mathematics or for employment in industry. HMC's Common Core provides mathematical biology majors with a strong multidisciplinary foundation, and the college offers many opportunities for students to engage in interdisciplinary research in biomathematics and quantitative biology. Students who choose this major become immersed in the scientific and intellectual cultures of both biology and mathematics, and the major is sufficiently flexible to allow students to concentrate in a particular area of mathematical biology. Students in this major have both a biology advisor and a mathematics advisor, who help them plan programs tailored to their interests and goals.

For more information (including course offerings), see <http://www.math.hmc.edu/program/>.

Since joining the department in the summer of 2001, I've been working to bring our systems up-to-date and make them easier to manage.

As of this writing, our department has settled on the CentOS rebuild of the Red Hat Enterprise Linux distribution. We're running CentOS 3 on the 60 or so Linux systems that the department has in use, and I'm looking at updating workstations to CentOS 4 in the summer of 2005.

In addition to a number of servers providing basic services to the department and individual faculty workstations, we have systems for use by students in the scientific-computing lab and the Mathematics Clinic lab.

In 2004, Professor Melissa O'Neill from the computer-science department and I put together a new Beowulf cluster (called Amber, after the books by Roger Zelazny) that we share with the CS department. It's currently in use by CS Clinic teams, some non-Mudd researchers, and Professor Belinda Thom's CS machine-learning class, and is available for use by researchers and students at Mudd and the other Claremont Colleges.

Our scientific-computing lab is also being used by classes in the CS department, as our systems are capable of running

software (such as MATLAB and Maple) that isn't available on the CS department's systems. I'm also proud to say that we have a growing reputation for providing stable services to our users, to the extent that some people use our machines rather than the systems their own departments provide.

In between all the work on hardware and operating systems, I've been working to create LaTeX classes for

typesetting senior theses, Clinic reports, and other documents. Our ICM/MCM class has been a hit with the judges of those competitions.

As this is the first issue of **MUDDmath** for a while, I'm not sure what future columns from me might include; any suggestions that you might have would be welcome. I can be reached at [cmc@math.hmc.edu](mailto:cmc@math.hmc.edu).

*Claire Connelly holds an undergraduate degree in anthropology, is a Debian Developer, and has been a GNU Emacs and MH user since 1990. She edited George Grätzer's First Steps in LaTeX in 1999 and the third edition of Math into LaTeX in 2000.*



## FunFact

### All Horses are the Same Color

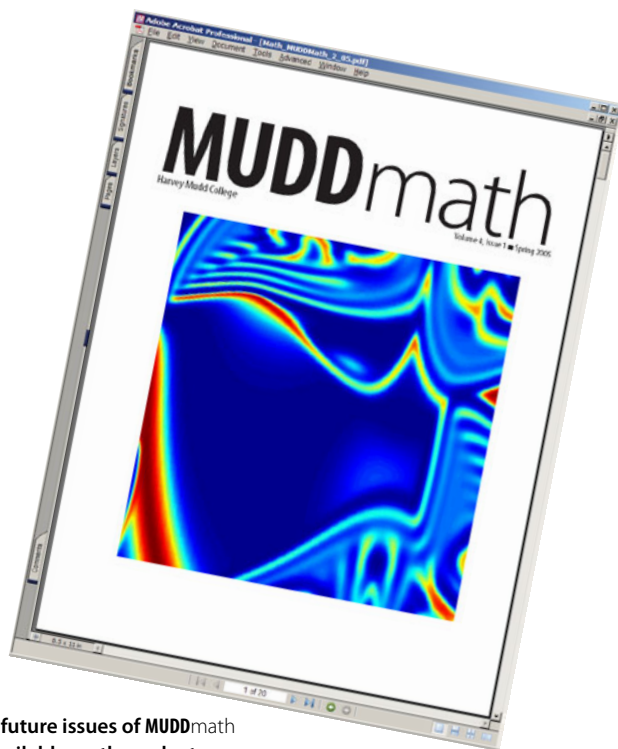
Since you Mudd alums all know how to prove things by induction, let's prove the following amazing fact.

**Theorem:** All horses are the same color.

**Proof:** We'll induct on the number of horses. The base case is clearly true. For the inductive step, assume any group of  $N$  horses have the same color, and consider any group of  $N+1$  horses. Excluding the last horse, we get a set of  $N$  horses. By the inductive hypothesis, these  $N$  horses all have the same color. But by excluding the first horse in the pack of  $N+1$  horses, we can conclude that the last  $N$  horses also have the same color. Therefore all  $N+1$  horses have the same color. QED.

What's going on here?! Clearly all horses don't have the same color. Can you figure out the flaw in the proof?

For more information and other Fun Facts, check out <http://www.math.hmc.edu/funfacts/>



This and future issues of **MUDDmath** will be available on the web at <http://www.math.hmc.edu/muddmath/>.

Teaching by a Beginning College or University Faculty Member. Francis also received the Hasse Prize from the MAA in 2001 for expository writing. In 2000, Art Benjamin received the MAA's Haimo Award for Distinguished College or University Teaching of Mathematics. Last summer, Professors Andrew Bernoff and Lesley Ward taught courses at the prestigious Park City Mathematics Institute. (Andy Bernoff was assisted by Jon Jacobsen and Darryl Yong '96.) Francis Su taught a course for them this past summer. In 2004, Lesley Ward taught harmonic analysis at the Program for Women in Mathematics, sponsored by the Institute for Advanced Study (IAS) and Princeton University. Darryl Yong also received a grant from IAS to expose high school and middle school teachers to interesting college-level mathematics that they can bring to the classroom.

Our research has been getting a lot of attention as well. Professor Weiqing Gu spoke at the International Congress of Mathematicians in 2002, held in Beijing. Andrew Bernoff and Francis Su each received NSF grants to support their research. In 2002, City College in New York hosted a conference entitled "Melvin Henriksen at 75: his research and co-workers."

Since 1999, we have sponsored our own mathematics research conference, called the Harvey Mudd College Mathematics Series (formerly called the Mount Baldy Conferences). Topics have been Analysis (organized by Professor Ward), Applied Mathematics (Bernoff and Hosoi), Differential Geometry (Gu), Applied Algebra and Combinatorics (Orrison) and Mathematical Biology (de Pillis and Jacobsen), and Geometry, Algebra, and Phylogenetic Trees (Su). These conferences have attracted students and faculty from Southern California and beyond. In 2005, the conference will be on Scientific Computing, organized by Darryl Yong.

In addition to the numerous research articles produced by our faculty, several books were written or edited by HMC faculty members, including "Introduction to Differential Equations, A Modeling Approach, 2nd Edition" by Borrelli and Coleman; "Polynomial Completeness for Algebraic Systems" by Alden Pixley and Kalle Kaarli; "Proofs that Really Count: The Art of Combinatorial Proof" by Arthur Benjamin and Jennifer Quinn; "Essays in Humanistic Mathematics" edited by Alvin White and "For All Practical Purposes, 6th Edition" edited by Francis Su. In addition, Al White served as editor of the Humanistic Mathematics Journal; Art Benjamin is co-editor of Math Horizons magazine, and Alfonso Castro is co-editor of the Electronic Journal of Differential Equations.

Other faculty highlights: In 1999, Lisette de Pillis built the first Beowulf Cluster at an undergraduate institution. The following year, she was the first woman from an undergraduate college to be honored as a Maria Goeppert-Mayer Distinguished Scholar at the Argonne National Labs. Hank Krieger, who coached the CMS men's tennis team from 1976–1999, received the Division III Coach of the Year (for the second time!) in 1993 from the Intercollegiate Tennis Association, and this year was inducted into the CMS Hall of Fame. Hank also served as Chair of the HMC faculty from 2001 to 2004.

Our Putnam team has done extremely well. We have had as many as seven students finish in the top 100 finishers on this exam, and our team has finished in the top 10 for the last three years. In 1991, HMC's team (Guy Moore, Tim Kokesh and Jon Leonard) finished third. HMC is the only undergraduate college to finish in the top five in the last 30 years, and has the highest per-capita student participation of any school in the country, thanks to Professors Andrew Bernoff and Francis Su who teach a weekly Putnam Seminar. In 1998, Andrew "RIF" Hutchings '98 won "College Jeopardy." Using some of the school's prize money and a fund raising effort led by Aaron Archer '98 and Patri Forwalter-Friedman '98, HMC raised \$75,000 to fund an endowed scholarship fund (the "RIF Fund") to reward and support students with exceptional promise on the Putnam exam. Andy Niedermaier '04 was the second recipient of the RIF Scholarship, and last year he, together with David Gaebler '04 and Jason Murcko '05, led HMC to a fifth place Putnam finish. As a result, HMC received \$5,000, most of which will be used to support future Putnam students and activities.

Although our Putnam record is impressive, no school has come close to our record of performance on the Mathematical Contest in Modeling (MCM) and the Interdisciplinary Contest in Modeling (ICM). In the history of these competitions, our student teams have won more *Outstanding* designations than any other school in the nation. In this year's ICM, from 143 participating teams, two of the four teams with the Outstanding designation were from HMC. Kudos to Michael Moody and Jon Jacobsen for organizing our teams.

Unrelated to the MCM or ICM, we should also mention the programming competition sponsored by the ACM (Association for Computing Machinery). In 1997, the all-math-major HMC team of Brian Carnes '97, Brian Johnson '98 and Kevin Watkins '98 finished first in this international competition, the only team from an undergraduate college to ever do so.

Other changes: At the end of the 2004–05 academic year, Hank Krieger will be retiring. We are looking to hire a new faculty member in the area of Statistics or Biostatistics. And finally, I am on sabbatical this year, and have handed over the reins of department chairmanship to Alfonso Castro. You'll be hearing from him in our next newsletter, which will hopefully arrive less than 15 years from now!

I'll conclude by mentioning a report of the external review team, who visited the department in the fall of 2002, after we had concluded a lengthy self-study. This team of mathematicians (from Williams, Rice, St. Olaf and UCSD) read our reports, interviewed our faculty, students and administration, evaluated our department, and "found the department one of the best at any liberal arts college or technical school in the country." We are flattered by this conclusion, and we will strive to continue to deserve that distinction in the future.

—Art Benjamin

## Robert James, Founding Mathematics Chair

*Professor Robert James, the founding chair of the HMC Department of Mathematics, passed away on Saturday September 25, 2004. Below are two remembrances of Bob—one from HMC Founding President Emeritus Joseph Platt and the other from Robin and Lori Ives '61.*

Bob James did a great job of building the mathematics program and the Department of Mathematics at Harvey Mudd College.

Bob did his undergraduate work at UCLA, where his father was professor of mathematics. (Father and son were co-authors of “The Mathematics Dictionary.”) During WWII, Bob, a Quaker, was a conscientious objector and spent his “service” as a firefighter in our local mountains, and as a guinea pig for various antimalarial drugs being considered for war use. When peace broke out, Bob took his doctorate in math at Caltech, and went for a postdoctoral year at Harvard as a Benjamin Peirce Fellow. He then joined the Berkeley faculty. Courtney Coleman took a course from him when Court was a Berkeley undergrad. The Oath controversy came along in the late 40s, and Bob refused to sign, moving to a faculty appointment at Haverford. That is where he was teaching when we met.

Herman Goldstine, a friend of mine who worked with von Neumann at the Institute for Advanced Study, knew that Bob had been offered a sabbatical fellowship at the Institute, that Bob was a first-rate mathematician, and that he also was well respected as a teacher at Haverford. I approached Bob (and Edith, his wife; they believed in family decisions) and the two decided to scrub the year at the Institute for Advanced Study in favor of helping to start a new college. I have been very grateful they did. Hence Bob was the one mathematician in our initial faculty of seven.

About 1974, Bob was asked to introduce a doctorate in mathematics into what is now Claremont Graduate University, and he founded that department also. He concluded that what CGU really needed was an applied mathematics program, and, he, with the help of Stavros Busenberg, got that program started. Bob left once that was started; he felt the new program needed an applied mathematician (which Bob was not) as chair. Bob moved to the Albany campus of the State University of New York, where he chaired that mathematics department, I think until his retirement.

You may gather I hold Bob in warm respect. That's right.

—Joe Platt

Robert James had a distinguished reputation in the mathematical community because of his contribution to the theory of Banach Spaces. He discovered an ingenious example that settled an important problem. James was a dedicated Quaker, a pacifist and a conscientious objector in World War II. In 1950, he resigned his appointment at the University of California at Berkeley because he refused to sign the loyalty oath that was then required by a California law.



From Berkeley, he moved to Haverford College, a Quaker college outside of Philadelphia. In his spare time there, apart from scholarly activities and teaching duties, he built a house on the college campus with his own hands, the assistance of his wife, Edith, and the help of students he hired as helpers.

He was hired to head the Department of Mathematics when Harvey Mudd College opened on a boulder field in Claremont, Calif. There he purchased an old house and, with his own labor, remodeled it extensively.

With the help of other members of the department, he wrote a rigorous textbook, entitled “University Mathematics,” that covered all the topics in Harvey Mudd College’s unusually wide ranging two-year course that was taught to all freshmen and sophomores. That textbook was

used for quite a few years until the decision was reached that it was too sophisticated to be appropriate for every student at the college.

However, very few students sold their copies of the text when the course was over; they kept it as a valuable reference.

He retired from Harvey Mudd College and moved to the Claremont Graduate School, where he established a program for an advanced degree in mathematics.

When James retired, he moved to Grass Valley in the foothills of the western Sierra Nevada, where he and Edith built yet another house for themselves.

—Robin and Lori Ives '61



## The Barbara Beechler I Knew

*Barbara Beechler, professor emerita at Pitzer College and former MAA Southern California Section Governor, died on March 18, 2003, at age 74 of complications due to cancer.*

I met Barbara Beechler when I joined the faculty of Harvey Mudd College in the fall of 1969. Barbara was the unique mathematician at Pitzer College and had come there two years earlier, and I soon learned that she was one of the most important members of the mathematical community in Claremont, in part because she worked harder than any two of the rest of us.

While few Pitzer College students take many courses in hard science, they still need to acquire mathematical skills even if they are not preparing to study calculus. Their needs are varied, no one individual could meet them all, and offering a few “remedial” courses could not possibly meet them. Barbara realized this very quickly and knew she had to help her students to make use of the resources available at The Claremont Colleges, as well as create special courses for different individuals. So, among other things, she familiarized herself with most of the mathematics courses offered at each of The Claremont Colleges as well as the teaching styles of each member of their faculties. When nothing else would work, she offered reading courses to students whose needs could not be met otherwise.

Understanding Barbara’s impact on The Claremont Colleges requires a little understanding of their nature at that time. They consisted of five completely independent undergraduate colleges and an independent graduate school, each with their own departmental structure, grading systems, deans, presidents and boards of trustees. Students enrolled in one college were allowed to take some of their courses at any of the others subject to complicated rules enforced capriciously in constant states of change. Sometimes parts of these institutions cooperate, and at other times, personality conflicts developed into institutional wars. This anarchy made Barbara’s formidable tasks even more difficult. Only a minority of her Pitzer colleagues had a friendly attitude towards mathematics, and she gained influence with her mathematical colleagues at the other colleges by offering help in a myriad of ways. Among other things, she took a lead in designing placement exams, she participated in programs to improve the teaching of mathematics in high school, she was an eagle-eyed proof reader who read the papers of colleagues for content as well as typography, she helped to make our colloquium series a success in part by entertaining speakers and attendees at her home at post-colloquium receptions, and she was the only person in Claremont who understood all of the grading systems at the different colleges. (Each college had a different one, and each student had to be graded by the system of the college in which he or she was enrolled. The easiest way to cope with this was to call Barbara and get a translation.) In this way, she became close to indispensable to all of us. She also worked hard at advising students and did more than her fair share of committee work at Pitzer.



To do all of this, Barbara saddled herself with an enormous workload, usually well in excess of 60 hours per week. She would often arrive at her office before 6:00 A.M., and not go home until dark, but she never felt she was doing enough. As the years passed, problems with her health developed, but even that did not slow her down. She “retired” from the Pitzer faculty in a formal way in 1989, but continued to teach and involve herself with mathematical activities in Claremont and with the Southern California MAA, where she saw to it that meetings were properly organized and became the secretary treasurer of the Section. When she took this office, the Section was badly in debt, but in a few years she built up a surplus. This was the result of her efficiency combined with doing herself many jobs that had been farmed out for many years at substantial cost to the Section. She became governor of the Section. She was awarded the Certificate of Meritorious Service at the 1995 national meetings, but the resulting recognition would not permit her to rest on her laurels. Her health continued to deteriorate, which slowed her down without stopping her.

The last few years of Barbara’s life were not happy ones. It was difficult for her to accept help from friends when her pleasure in life had come mostly from being useful to others. Only death could stop her, but even that has not eradicated our memories of someone who gave so much for so little.

—Melvin Henriksen

*This article first appeared in the February 2004 issue of the Southern California-Nevada Section Mathematical Association of America Newsletter.*

# WHAT'S NEW?

**WE** would  
like to **HEAR**  
from **YOU!**

Please drop us a line to tell us what you're up to. We are especially interested in news suitable for sharing in the "Alumni News" section of forthcoming issues of **Muddmath**.

Name

Year of Graduation

Email

Permission to share your information in the **Muddmath** Newsletter—please sign and date

Current job title and employer

Professional and/or personal news:

☐

I would like to order an HMC Mathematics Absolute Value T-Shirt.

Size:      S              M              L              XL

I have enclosed a check for \_\_\_\_\_ (\$15 each shirt).

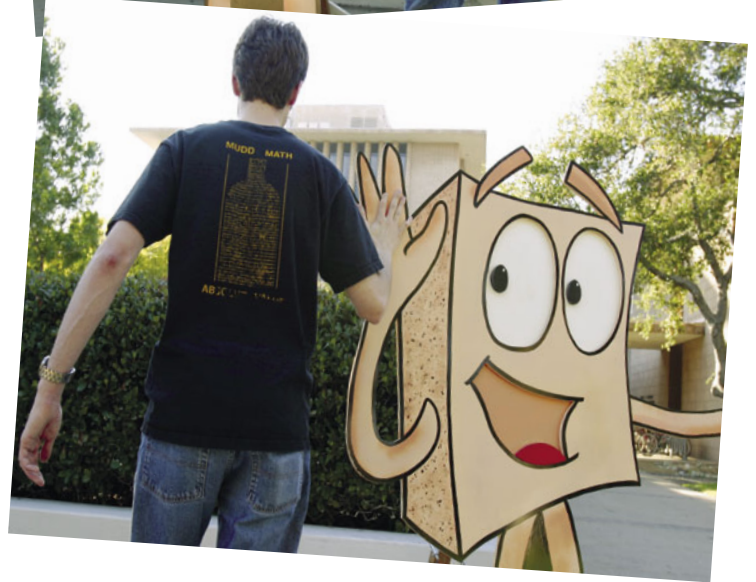
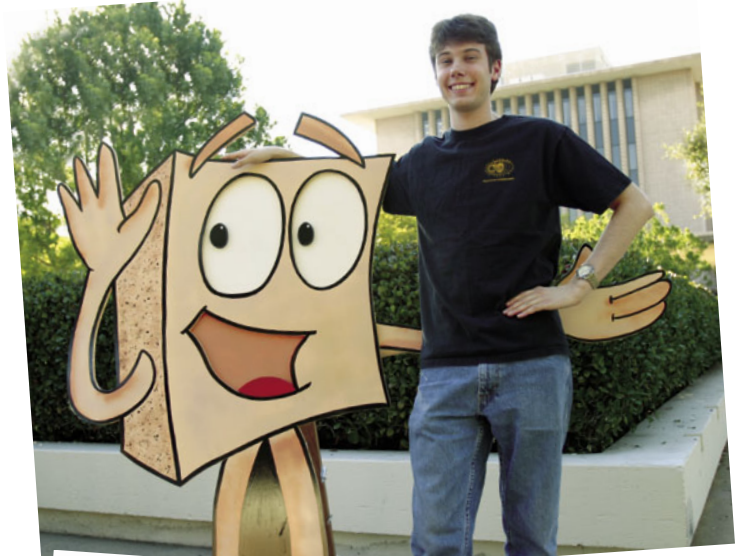
**Please return this form to:**

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**Harvey Mudd College, 301 Platt Boulevard, Claremont, CA 91711**

# MUDD MATH

$$\begin{aligned} \lim_{x \rightarrow a} f(x) &= L \Leftrightarrow \\ (\forall \epsilon > 0) (\exists \delta > 0) & \text{ if } 0 < |x - a| < \delta \text{ then } |f(x) - L| < \epsilon \\ \lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n &= e \\ e \cdot e^{i\pi} &= -1 \cdot f'(a) = \\ \lim_{h \rightarrow 0} [f(a+h) - f(a)]/h &= (fg)'(x) = \\ f'(x)g(x) + f(x)g'(x) \\ \int - (fg)'(x) dx / g^2(x) &= (fg) \cdot \\ \int (x-a) \cdot f_a^b f(x) dx &= \lim_{n \rightarrow \infty} \sum_{k=1}^n \\ \int_a^b f(x) \cdot f_a^b f(x) dx &= F(b) - F(a) \cdot A = \frac{1}{2} f_a \\ \sqrt{1 + f'(x)^2} dx \cdot S &= 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx \cdot \\ f(x)^2 dx \cdot \ln x &= \int_1^x \frac{1}{t} dt \cdot (e^x)' = e^x \cdot uv = \int f' uv' dx + \\ f' uv - f(u)'(x) &= \frac{1}{n} x^n + \dots + \frac{1}{n} x^n + \dots \ln 2 : \\ \frac{n+1}{(n-1)!} (x-a)^{n+1} \cdot e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{1}{n!} x^n + \dots \ln 2 : \\ -\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots & \cdot (1+x)^2 = \sum_{k=0}^{\infty} \binom{2}{k} x^k \cdot (u \times v)' = \\ u' \times v + u \times v' & \cdot s(t) = \int_a^t \|r'(u)\| du \cdot \kappa = \frac{|r''(u)|}{|r'|^3} \cdot \tau = \\ r' \times r'' \cdot r''' \cdot \frac{d\tau}{ds} &= \kappa N \cdot \frac{d\tau}{ds} = \tau N \cdot \frac{dN}{ds} = -\kappa \tau \cdot r B \cdot D(f \circ g) = \\ Df \circ g Dg \cdot f'(r(t)) &= \nabla f(r(t)) \cdot r'(t) \cdot S = \iint_R |T_u(u, v) \times \\ T_v(u, v)| dA \cdot \iint_R \phi(x, y) dx dy &= \iint_S \phi(f(u, v), g(u, v)) \frac{\partial(f, g)}{\partial(u, v)} du dv \cdot \\ f_C P dx + Q dy &= \iint_R (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}) dA \cdot \nabla \cdot (\nabla \times F) = 0 \cdot \iint_S (\nabla \times \\ F) \cdot ndS = \iint_S F \cdot \iint_V \nabla \cdot F dV &= \iint_{\partial V} F \cdot ndS \cdot \int_{\partial D} w = \\ \int_D w \cdot dw \cdot |a_1 + a_2 + \dots + a_n| &\leq |a_1| + |a_2| + \dots + |a_n| \cdot (Av, u) \times \\ (v, A^* u) \cdot \det A &= \sigma \in S_n \cdot (-1)^{\alpha_1 \alpha_2 \dots \alpha_n} \dots a_{\alpha_1 \alpha_2 \dots \alpha_n} \cdot Av = \lambda v \cdot \\ (v, A^* u) \cdot \det A &= 1/\det A^{-1} \cdot (AB)^* = B^* A^* \cdot \sigma(A) + n(A) \cdot A = \\ C^{-1} TC \cdot \det A &= 1/\det A^{-1} \cdot (AB)^* = B^* A^* \cdot o(A) \cdot a \in G \Rightarrow \\ \dim A \cdot \|(v, w)\|^2 &\leq \|(v)\|^2 \|(w)\|^2 \cdot a \in G \Rightarrow o(A) \cdot o(G/N) = \\ a \circ(G/N) = e \cdot a \circ(G/N) &= 1 \bmod n \cdot o(HK) = \frac{o(H)o(K)}{o(H \cap K)} \cdot o(G/N) = \\ o(G)/o(N) \cdot \Gamma(\nu) &= \int_0^\infty e^{-t} t^{\nu-1} dt \cdot \Gamma(\nu+1) = \nu \Gamma(\nu) \cdot \frac{1}{2} \int (f(x + \\ o(G)) + o(N)) &= \frac{1}{2\pi} \int_0^{2\pi} f(\xi) d\xi + \frac{1}{2} \sum_{n=1}^\infty \frac{1}{n} \int_0^{2\pi} f(\xi) \cos n(\xi - x) d\xi \cdot \\ 0) + f(x - 0) &= \frac{1}{2\pi} \int_0^{2\pi} f(\xi) d\xi + \frac{1}{2} \sum_{n=1}^\infty \frac{1}{n} \int_0^{2\pi} f(\xi) \cos n(\xi - x) d\xi \cdot \\ f(x) &= \frac{1}{2\pi} \int_0^{2\pi} f(\xi) \cos \alpha(\xi) d\xi + \frac{1}{2} a_0^2 + \sum_{n=1}^\infty (a_n^2 + \\ b_n^2) &= \frac{1}{2\pi} \int_0^{2\pi} f^2(x) dx \cdot \rho^2 y' + \rho y + (x^2 \rho^2 - \nu^2) y = 0 \cdot J_n(x) = \\ \sum_{j=0}^\infty \frac{(-1)^j}{j! (j+1)!} (\frac{x}{2})^{j+2j} \cdot (x^{-n} J_n(x))' &= -x^{-n} J_{n+1}(x) \cdot J_n(x) = \\ \sum_{j=0}^\infty \frac{(-1)^j}{j! (j+1)!} (\frac{x}{2})^{j+2j} \cdot (x^{-n} J_n(x))' &= -x^{-n} J_{n+1}(x) \cdot J_n(x) = \\ \frac{1}{2\pi} \int_0^{2\pi} (\cos(n\phi - x \sin \phi) d\phi \cdot (1 - x^2)^{n-1} &= 2ny' + \lambda y = 0 \cdot P_n(x) = \\ \frac{1}{2\pi} \sum_{j=0}^\infty \frac{(-1)^j}{j! (j+1)!} (\frac{x}{2})^{j+2j} \cdot (x^{-n} J_n(x))' &= -x^{-n} J_{n+1}(x) \cdot J_n(x) = \\ v_y, u_y &= -v_x \cdot \log z = \text{Log} z + i\theta \cdot f(z) = \frac{1}{2\pi i} \int_C \frac{f(\zeta) d\zeta}{(\zeta - z)^{n+1}} + \\ \dots + (z - z_0)^n \int_C \frac{f(\zeta) d\zeta}{(\zeta - z_0)^{n+1}} + \sum_{n=0}^\infty \frac{1}{(z - z_0)^{n+1}} \int_C \frac{f(\zeta) d\zeta}{(\zeta - z_0)^{n+1}} \cdot K_1 + \\ \dots + K_n &= \frac{1}{2\pi i} \int_C f(z) dz \times u(r, \theta) \cdot f(z) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{f(r_0 - 2r_0 \cos(\theta - \theta_0) + r_0^2)}{r_0^2 - 2r_0 \cos(\theta - \theta_0) + r_0^2} d\theta \cdot \\ \pi^2/6 &= 1 + (1/2)^2 + (1/3)^2 + \dots \cdot \pi = 2 \frac{2}{3} \frac{3}{4} \frac{4}{5} \frac{5}{6} \dots \cdot \pi = \\ &= 4(1 - 1/3 + 1/5 - 1/7 + \dots) \cdot \pi = 6(1 \frac{1}{2} + \frac{1}{3 \cdot 2^3} + \frac{1}{4 \cdot 2^4} + \dots) \cdot \frac{\pi}{2} = \\ &= \frac{2^2-1}{2^2-1} \frac{3^2-1}{3^2-1} \frac{5^2-1}{5^2-1} \frac{7^2-1}{7^2-1} \dots \cdot \det A = \sigma \in S_n \cdot (-1)^{\alpha_1 \alpha_2 \dots \alpha_n} \dots a_{\alpha_1 \alpha_2 \dots \alpha_n} \cdot \\ Av = \lambda v \cdot A = C^{-1} TC \cdot \det A &= 1/\det A^{-1} \cdot (AB)^* = B^* A^* \cdot \\ |a_1| + |a_2| + \dots + |a_n| \leq |a_1| + |a_2| + \dots + |a_n| & \cdot \|(v, w)\|^2 \leq \|(v)\|^2 \|(w)\|^2 \cdot$$



**ABSOLUTE VALUE**—that's what Mudder math major Eric Malm '05 and his friend, Wally Mudd, think about the Math T-shirt as Eric models the front and back for Wally! A "High Five" to them both.

# ABSOLUTE VALUE

You can purchase a Mudd Math Absolute Value T-Shirt by filling out the order form on the previous page. The T-shirt is navy blue and gold with the Harvey Mudd College seal on the front and Michael Moody's famous Absolute Value design on the back.

# MUDD MATH

$$\begin{aligned}
 & \lim_{x \rightarrow a} f(x) = L \Leftrightarrow (\forall \epsilon > 0)(\exists \delta > 0)(\text{if } 0 < |x - a| < \delta \text{ then } |f(x) - L| < \epsilon) \bullet \\
 & \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e \bullet e^{i\pi} = -1 \bullet f'(a) = \lim_{h \rightarrow 0} [f(a+h) - f(a)]/h \bullet (fg)'(x) = f'(x)g(x) + f(x)g'(x) \\
 & -(fg')(x) \bullet [f^2(x)] \bullet (f \circ g)'(x) = f'(x) \bullet \int_a^b f(x) dx = \lim_{\|P\| \rightarrow 0} \sum_{i=1}^n f(x_i^*) \bullet \int_a^b f(x) dx = F(b) - F(a) \bullet A = \frac{1}{2} \int_{-\alpha}^{\alpha} \sqrt{1 + f'(x)^2} dx \bullet S = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx \bullet \\
 & f(x)^2 dx \bullet \ln x = \int_1^x \frac{1}{t} dt \bullet (e^x)' = e^x \bullet uv = \int uv' dx + \int vu' \bullet \frac{\sin x}{x} dx = \frac{\pi}{2} \bullet f(x) = f(a) + f'(a)(x-a) + \dots + \frac{f^{(n)}(a)}{n!} (x-a)^n \bullet \\
 & \frac{(n+1)!}{(n+1)!} (x-a)^{n+1} \bullet e^x = 1 + x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \dots + \frac{1}{n!} x^n + \dots \bullet \ln 2 = -\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \bullet (1+x)^s = \sum_{k=0}^{\infty} \binom{s}{k} x^k \bullet (u \times v)' = u' \times v + u \times v' \bullet s(t) = \int_a^t \|r'(u)\| du \bullet \kappa = \frac{|r' \times r''|}{|r'|^3} \bullet \tau = \frac{r' \times r'' \cdot r''}{|r'| \times |r''|^2} \bullet \frac{dT}{ds} = \kappa N \bullet \frac{dB}{ds} = \tau N \bullet \frac{dN}{ds} = -\kappa T - \tau B \bullet D(f \circ g) = Df \circ g Dg \bullet f'(r(t)) = \nabla f(r(t)) \cdot r'(t) \bullet S = \iint_R |T_u(u, v) \times T_v(u, v)| dA \bullet \iint_R \phi(x, y) dx dy = \iint_S \phi(f(u, v), g(u, v)) \frac{\partial(f, g)}{\partial(u, v)} du dv \bullet \\
 & \oint_C P dx + Q dy = \iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA \bullet \nabla \cdot (\nabla \times F) = 0 \bullet \iint_S (\nabla \times F) \cdot ndS = \iint_{\partial S} F \cdot dr \bullet \iiint_V \nabla \cdot F dV = \iint_{\partial V} F \cdot ndS \bullet \int_{\partial D} \omega = \int_D d\omega \bullet |a_1 + a_2 + \dots + a_n| \leq |a_1| + |a_2| + \dots + |a_n| \bullet (Av, w) = (v, A^* w) \bullet \det A = \sum_{\sigma \in S_n} (-1)^\sigma a_{1\sigma(1)} a_{2\sigma(2)} \dots a_{n\sigma(n)} \bullet Av = \lambda v \bullet A = C^{-1} TC \bullet \det A = 1/\det A^{-1} \bullet (AB)^* = B^* A^* \bullet r(A) + n(A) = \dim A \bullet |(v, w)|^2 \leq \|v\|^2 \|w\|^2 \bullet a \in G \Rightarrow o(a) \mid o(G) \bullet a \in G \Rightarrow a^{o(G)} = e \bullet a^{\phi(n)} \equiv 1 \pmod{n} \bullet o(HK) = \frac{o(H)o(K)}{o(H \cap K)} \bullet o(G/N) = o(G)/o(N) \bullet \Gamma(\nu) = \int_0^\infty e^{-t} t^{\nu-1} dt \bullet \Gamma(\nu+1) = \nu \Gamma(\nu) \bullet \frac{1}{2} [f(x+0) + f(x-0)] = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\xi) d\xi + \frac{1}{\pi} \sum_{n=1}^{\infty} \int_{-\pi}^{\pi} f(\xi) \cos n(\xi-x) d\xi \bullet f(x) = \frac{1}{\pi} \int_0^\infty \int_{-\infty}^\infty f(\xi) \cos[\alpha(\xi-x)] d\xi d\alpha \bullet \frac{1}{2} a_0^2 + \sum_{n=1}^\infty (a_n^2 + b_n^2) = \frac{1}{\pi} \int_{-\pi}^{\pi} f^2(x) dx \bullet \rho^2 y'' + \rho y' + (\lambda^2 \rho^2 - \nu^2) y = 0 \bullet J_\nu(x) = \sum_{j=0}^\infty \frac{(-1)^j}{j! \Gamma(\nu+j+1)} \left(\frac{x}{2}\right)^{\nu+2j} \bullet (x^{-n} J_n(x))' = -x^{-n} J_{n+1}(x) \bullet J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\phi - x \sin \phi) d\phi \bullet (1-x^2)y'' - 2xy' + \lambda y = 0 \bullet P_n(x) = \frac{1}{2^n} \sum_{j=0}^n \frac{(-1)^j}{j!} \frac{(2n-2j)!}{(n-j)!} x^{n-2j} \bullet z^n = r^n (\cos n\theta + i \sin n\theta) \bullet u_x = v_y, u_y = -v_x \bullet \log z = \text{Log } r + i\theta \bullet f(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z)}{z-z_0} dz \bullet 2\pi i f(z) = \sum_{n=0}^\infty (z-z_0)^n \int_{C_1} \frac{f(z') dz'}{(z'-z_0)^{n+1}} + \sum_{n=0}^\infty \frac{1}{(z-z_0)^n} \int_{C_2} \frac{f(z') dz'}{(z'-z_0)^{1-n}} \bullet K_1 + \dots + K_n = \frac{1}{2\pi i} \int_C f(z) dz \bullet u(r, \theta) = \frac{1}{2\pi} \int_0^{2\pi} \frac{(r_0^2 - r^2) u(r_0, \theta')}{r_0^2 - 2r_0 r \cos(\theta' - \theta) + r^2} d\theta' \bullet \pi^2/6 = 1 + (1/2)^2 + (1/3)^2 + \dots \bullet \pi = \frac{2^2}{1} \frac{2}{3} \frac{4}{5} \frac{6}{7} \dots \bullet \pi = 4(1 - 1/3 + 1/5 - 1/7 + \dots) \bullet \pi = 6(\frac{1}{2} + \frac{1}{2 \cdot 3 \cdot 2^5} + \frac{1}{2 \cdot 4 \cdot 5 \cdot 2^5} + \dots) \bullet \frac{\pi^2}{6} = \frac{2^2}{2^2-1} \frac{3^2}{3^2-1} \frac{5^2}{5^2-1} \frac{7^2}{7^2-1} \dots \bullet \det A = \sum_{\sigma \in S_n} (-1)^\sigma a_{1\sigma(1)} a_{2\sigma(2)} \dots a_{n\sigma(n)} \bullet Av = \lambda v \bullet A = C^{-1} TC \bullet \det A = 1/\det A^{-1} \bullet (AB)^* = B^* A^* \bullet |a_1 + a_2 + \dots + a_n| \leq |a_1| + |a_2| + \dots + |a_n| \bullet |(v, w)|^2 \leq \|v\|^2 \|w\|^2 \bullet
 \end{aligned}$$

## ABSOLUTE VALUE

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